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ABSTRACT

This is one of eighteen sets of individualized mathematics problems developed by the Oregon Vo-Tech Math Project. Each of these problem packages is organized around a mathematical topic and contains problems related to diverse vocations. Solutions are provided for all problems. Problems in which calculus and statistics are applied to forestry, manufacture of forest products, and electronics are presented. Problems call for computation of derivatives, integrals, means, and standard deviations. (SD)

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I
L INDIVIDUALIZED
LEARNING
S SYSTEMS

Individualized Math Problems in

Calculus & Statistics

Oregon Vo-Tech Mathematics Problem Sets

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Individualized Learning Systems

MATHEMATICS

Calculus & Statistics

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A. Problem with Solution

1. A forest technician made the following randomly selected diameter measurements in connection with a proposed timber sale. 20 in, 30 in, 25 in, 10 in and 40 in.

   Determine a) the mean diameter, b) the standard deviation, c) the coefficient of variation, d) the spread of diameters at 1 standard deviation and e) assuming the total number of trees involved is greater than 1,000, would you consider the above sample large enough to make any useful statistical calculations?

   Given: Standard Deviation $= \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}}$

   Solution:

   a) Mean Diameter, $\bar{d} = \frac{\sum d}{n}$

   $= \frac{20 \text{ in} + 30 \text{ in} + 25 \text{ in} + 10 \text{ in} + 40 \text{ in}}{5}$

   $= 25 \text{ in}$

   b) Standard Deviation $= \sqrt{\frac{\sum d^2 - \frac{(\sum d)^2}{n}}{n-1}}$

   $= \sqrt{\frac{3{,}625 \text{ sq in} - 3{,}125 \text{ sq in}}{4}}$

   $= 11.18 \text{ in}$

   c) Coefficient of Variation $= \frac{\text{Standard Deviation}}{\text{Mean Diameter}}$

   $= \frac{11.2 \text{ in}}{25 \text{ in}} = 0.448$

   d) Spread of Diameters $= 25 \text{ in} \pm 11.2 \text{ in}$
   $= 13.8 \text{ in}$ to 36.2 in

   e) The sample is too small.

B. Problem without Solution

2. The diameters of 10 randomly selected trees are given below. Determine the arithmetic mean diameter and the standard deviation.

Tree #	1	2	3	4	5	6	7	8	9	10
Diameter (d.i.b.)	9"	9"	11"	9"	7"	7"	10"	8"	9"	11"

2. Arithmetic mean $= \frac{9" + 9" + 11" + 9" + 7" + 7" + 10" + 8" + 9" + 11"}{10}$

$= \frac{90 \text{ in}}{10} = 9 \text{ in}$

Standard deviation $= \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n - 1}}$

$= \sqrt{\frac{828 \text{ sq in} - 810 \text{ sq in}}{9}}$

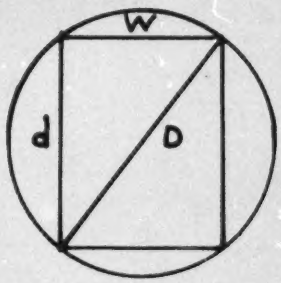
$= \sqrt{\frac{18 \text{ sq in}}{9}}$

$= \sqrt{2 \text{ sq in}}$

$= 1.414 \text{ in}$

A. Problem with Solution

1. The stiffness of a rectangular beam is proportional to the width, W, and the cube of the depth, d. Determine W and d in terms of the diameter of the stiffest beam which can be cut from a log of diameter D. Assume the log has the form of a right circular cylinder.



Solution:

$$S = KW(d^3)$$

Also $D^2 = W^2 + d^2$ by Pythagorean Theorem

$$\therefore S = d^3\sqrt{D^2 - d^2}$$

$$\frac{dS}{dd} = \frac{d^3(-2d)}{2\sqrt{D^2 - d^2}} + 3d^2\sqrt{D^2 - d^2}$$

$$= \frac{3D^2d^2 - 4d^4}{\sqrt{D^2 - d^2}}$$

Setting this expression equal to zero provides 4 solutions for d. $d = 0, 0, \pm\frac{\sqrt{3}\,D}{2}$

However since $0<d<D$,

$$d = \frac{\sqrt{3}\,D}{2}$$

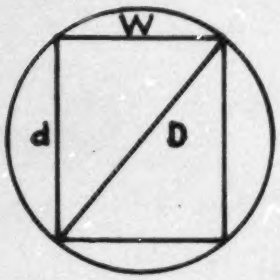
$$W = \sqrt{D^2 - d^2} = \sqrt{D^2 - \frac{3D^2}{4}} = \frac{D}{2}$$

Note: The fact that the above solution provides a maximum strength beam is obvious from the nature of the problem, however the second derivative or other conventional tests may be employed if a mathematical proof is considered necessary.

B. Problem without Solution

2. Given that the strength S of a rectangular beam is proportional to the width W, and the depth d, determine the relation of the width and depth to the diameter of the strongest timber that can be cut from a log of diameter D. Assume, of course, that the log is round and has no taper.

2.



Given the strength, $S = KWd$

Also $D^2 = W^2 + d^2$ by Pythagorean Theorem

$S = KW(D^2 - W^2)$
$S = KWD^2 - KW^3$

$\frac{dS}{dW} = KD^2 - 3KW^2$

Set $\frac{dS}{dW} = 0$

Then $0 = KD^2 - 3KW^2$

$K \neq 0$

Then $D^2 - 3W^2 = 0$

$$W = \frac{\sqrt{3}\,D}{3}$$

Since $D^2 = W^2 + d^2$
$d^2 = D^2 - W^2$
$d = \sqrt{D^2 - W^2}$
$d = \sqrt{D^2 - \frac{D^2}{3}}$
$d = \sqrt{(2/3)D^2}$
$d = \frac{\sqrt{6}\,D}{3}$

Note: $\frac{d_2S}{dW_2} < 0$ therefore these are maximum values

A. Problems with solutions.

1. A sample plot contains 21,780 sq. ft. of area and the following Douglas Fir trees:

DBH	No. trees	DBH	No. trees
2	3	12	3
4	2	14	18
6	4	16	10
8	1	18	22
10	6	20	15

a) determine the weighted average stand diameter.
b) determine basal area/ acre ($A = \pi r^2$)

Solution:

a) $$\text{avg} = \frac{2"(3)\text{tree} + 4"(2)\ \text{tree} + 6"(4)\text{tree} + 8"(1)\text{tree} + 10"(6)\text{tree} +}{3\ \text{tree} + 2\ \text{tree} + 4\ \text{tree} + 1\ \text{tree} + 6\ \text{tree} +}$$

$$+ \frac{12"(3)\ \text{tree} + 14"\ (18)\text{tree} + 16"\ (10)\text{tree} + 20"(15)\text{tree}}{+ 3\ \text{tree} + 18\ \text{tree} + 10\ \text{tree} + 22\ \text{tree} + 15\ \text{tree}} =$$

$$= \frac{1250\ \text{tree-in}}{84\ \text{tree}} = 14.9 \text{ inches.}$$

b) since 21,780 sq. ft. = 1/2 acre we multiply our result by 2.

the area of a 14.9" diameter tree is:

$$A = (\pi)\left(\frac{14.9\ \text{in}}{2}\right)^2 = (3.1416)\ (7.45\ \text{in})^2 = 174.4 \text{ sq. in.}$$

$$\text{or} = (174.4\ \text{sq. in.})\ \left(\frac{1\ \text{sq. ft}}{144\ \text{sq. in.}}\right) = 1.21 \text{ sq. ft.}$$

$$\left(\frac{1.21\ \text{sq ft}}{\text{tree}}\right)\left(\frac{84\ \text{trees}}{\text{plot}}\right)\left(\frac{2\ \text{plots}}{\text{acre}}\right) = 203.3 \text{ sq. ft / acre}$$

A. Problems with solutions, continued.

2. The total value of each of 18 logging loads going into a mill were: $80, $72, $77, $110, $113, $120, $107, $121, $125, $109, $96, $123, $102, $117, $169, $85, $105 and $112. Determine a) mean, b) standard deviation, c) coefficient of variation, d) range of values which should contain 68% of data, e) standard error of mean, f) the true average value/load will fall within what range of 68 out of 100 times?

Solutions:

a) $\bar{x} = \frac{\Sigma x}{n} = \frac{\$1943}{18} = \$107.94$

b) $(SD) = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n-1}} = \sqrt{\frac{(\$467.09)^2 - \frac{(\$1943)^2}{18}}{18 - 1}} = \sqrt{\frac{(467.09)^2 - (457.97)^2}{18-1}}$

$= \sqrt{(\$22.28)^2} = \$22.28$

c) $C = \frac{(SD)\ (100)}{\bar{x}} = \frac{\$22.28\ (100)}{\$107.94} = 20.64\%$

d) 68% of the data will fall within one standard deviation of the mean or in this case between:

$107.94 - $22.22 and $107.94 + $ 22.28 or between $85.66 and $130.22

e) $SE = \frac{SD}{\sqrt{n}} = \frac{\$22.28}{\sqrt{18}} = \frac{\$22.28}{4.243} = \$\ 5.25$

f) 68 out of 100 times the true average value/load will fall within one standard error of the completed mean or in this case between

$107.94 - $5.25 and $107.94 + $5.25 or $102.69 and $113.19

B. Problems without complete solutions.

3. In a timber sale there were: 1) 1421 merchantable ponderosa pine each with an average DBH (diameter at breast height) of 21 inches, an average gross volume of .345 M bd-ft (million board feet) an average number of logs of 3.07 and an average scaling defect of 10%; and 2) 706 Douglas fir each with an average DBH of 25 inches, and average gross volume of .746 M bd-ft, an average number of logs of 4.33, and an average scaling defect of 4%. Determine :

   a) the average DBH of all trees in the sale.
   b) the average gross volume per tree in the sale.
   c) the average number of logs per tree in the sale.
   d) the average per cent scaling defect for all trees in the sale.

4. The total value of each of 20 loads of timber at a scaling station were: $112, $106, $114, $97, $92, $120, $ 146, $133, $147, $63, $93, $78, $129, $75, $116, $105, $70, $76, $54, $84. Determine:

   a) mean
   b) standard deviation
   c) coefficient of variation
   d) standard effor of mean.

B. Problems without complete solutions, continued.

5. Assume ten 1/5 acre plots have been measured on a "forty". The net volumes in million of board-feet (M bd.ft) for the plots are: 8.5, 9.2, 13.6, 19.4, 10.8, 11.0, 15.4, 20.3, 8.9, 10.0. For the above cruise determine the following:

   a) mean
   b) standard deviation
   c) coefficient of variation
   d) 68% of the data should fall within what two values?
   e) standard error of the mean
   f) the true average volume of the entire stand will fall within what two values, 68 out of 100 times?

6. Twenty-five trees have been chosen which will yield the following board footage: 630, 3450, 1400, 950, 50, 800, 7050, 1750, 90, 1680, 180, 2700, 420, 7000, 6600, 130, 90, 370, 440, 630, 400, 540, 1100, 1900, 1600. For this cruise determine:
   a) mean, b) standard deviation, c) coefficient of variation d) what range should contain 68% of data? e) standard error of mean, f) the true average volume of the stand will fall within what two values, 68 out of 100 times?

Complete solutions to B problems.

3. Solutions:

a) $\bar{x} = \frac{1421\text{trees}\ (21)" + 706\text{trees}(25)"}{1421\text{trees} + 706\text{trees}} = \frac{29841\ \text{intree} + 17650\ \text{intree}}{2127\ \text{trees}} =$

$\frac{47491\ \text{tree in}}{2127\ \text{tree}} = 22.3\ \text{inches.}$

b) $\bar{x} = \frac{1421\text{trees}\ (.345\text{MBF}) + 706\text{trees}(.746\text{MBF})}{1421\text{trees} + 706\text{trees}} = \frac{490.245\text{MBFtree}}{2127\ \text{trees}} =$

$\frac{526.676\ \text{MBFtree}}{2127\ \text{tree}} = \frac{1016.921\text{MBFtree}}{2127\ \text{tree}} = .478\ \text{MBF}$

c) $\bar{x} = \frac{1421\text{trees}(3.07\text{logs}) + 706\text{trees}(4.33\text{log})}{1421\ \text{tree} + 706\ \text{tree}} = \frac{4362.47\text{log tree} +}{2127\ \text{trees}}$

$\frac{3056.98\ \text{log tree}}{2127\ \text{tree}} = \frac{7419.45\ \text{log tree}}{2127\ \text{tree}} = 3.49\ \text{logs}$

d) $\bar{x} = \frac{1421\text{tree}\ (.10) + 706\text{tree}(.04)}{1421\ \text{tree} + 706\ \text{tree}} = \frac{142.1\text{tree} + 28.24\ \text{tree}}{2127\ \text{tree}} =$

$\frac{170.34\ \text{tree}}{2127\ \text{tree}} = (.08)(100) = 8\%$

4. Solutions:

a) $\bar{x} = \frac{\Sigma x}{n} = \frac{\$2010}{20} = \$100.50$

b) $SD = \sqrt{\frac{x - \frac{x}{n}}{n-1}} = \sqrt{\frac{(\$464.50)^2 - \frac{(\$2010)^2}{20}}{}} =$

$\sqrt{\frac{(\$464.50)^2 - (\$449.45)^2}{19}} = \sqrt{\frac{(\$117.28)^2}{19}} =$

$= \sqrt{(\$26.90)^2} = \$26.90$

c) The coefficient of variation is simply the standard deviation expressed as a per cent of the mean so,

$c = \frac{(SD)\ (100)}{\bar{x}} = \frac{(\$26.9)\ (100)}{\$100.50} = 26.77\%$

D) $SE = \frac{SD}{\sqrt{n}} = \frac{\$26.9}{\sqrt{20}} = \frac{\$26.9}{4.472} = \$6.02$

Complete solutions to B problems, continued.

5. Solution:

a) $\bar{x} = \frac{\Sigma x}{n} = \frac{127.1 \text{ MBF}}{10} = 12.7 \text{ M bd ft.}$

b) $SD = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{N}}{n - 1}} = \sqrt{\frac{(42.24 \text{ mbf})^2 - \frac{(127.1)^2}{10}}{9}} = \sqrt{\frac{(42.24\text{mbf})^2 - (40.19\text{mbf})^2}{9}}$

$= \sqrt{\frac{(12.99 \text{ mbf})^2}{9}} = \sqrt{(4.33 \text{ mbf})^2} = 4.33 \text{ mbf}$

c) The coefficient of variation is simply the standard deviation expressed as a per cent of the mean so,

$c = \frac{(SD)\ (100)}{\bar{x}} = \frac{(4.33\text{mbf})(100)}{12.7 \text{ mbf}} = 34.09\%$

d) 68% of the data will fall within one standard deviation of the mean or in this case between:

12.7 mbf + 4.33 mbf and 12.7 mbf - 4.33 mbf or, 17.03mbf and 8.37mbf.

e) $SE = \frac{SD}{\sqrt{n}} = \frac{4.33 \text{ mbf}}{\sqrt{10}} = 1.37 \text{ M Bd ft.}$

F) 68 out of 100 times the true average volume will fall within one standard error of the computed mean or in this case between:

12.7 mbf - 1.37 mbf and 12.7 mbf + 1.37 mbf or 11.33 mbf and 14.07mbf.

Complete solutions to B problems, continued.

6. Solution:

a) $\bar{x} = \frac{\Sigma x}{n} = \frac{41950 \text{ bd ft}}{25} = 1678 \text{ Bd ft.}$

b) $SD = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n-1}} = \sqrt{\frac{(13{,}418 \text{ bd-ft})^2 \; (41950 \text{ bd-ft})^2}{25-1} \; \frac{}{25}}$

$= \sqrt{\frac{(13{,}418 \text{ bd-ft})^2 - (8{,}390 \text{ bd-ft})^2}{24}} = \sqrt{\frac{(10{,}471.42 \text{ bd-ft})^2}{24}}$

$= \sqrt{(2{,}137.47 \text{ bd ft})^2} = 2137 \text{ Bd-ft.}$

c) $c = \frac{(SD)(100)}{\bar{x}} = \frac{(2137 \text{ bd-ft})\ (100)}{1678 \text{ bd ft}} = 127.4\%$

d) 1678 bd ft - 2137 bd ft or 0 to 1678 bd ft + 2137 bd ft = 3815 bd ft.

(mean $\pm$ one st deviation).

e) $\frac{SD}{\sqrt{n}} = \frac{2137 \text{ bd ft}}{\sqrt{25}} = \frac{2137 \text{ bd ft}}{5} = 427.4 \text{ Bd ft}$

f) The true average volume of the stand will fall within one standard error if the computed mean 68 out of 100 times.

1678 bd ft - 427.4 bd ft = .250.6 bd ft to 1678 bdft + 427.4bdft = 2105.4 bdft.

A. Problems with Solutions

1. The voltage dropped by an inductor, L, is a function of the rate of change of current through the inductor as follows:

$v_L = L \frac{di}{dt}$ Where, V_L = Voltage dropped across the inductor

L = Inductance in Henrys
i = Current in Amperes
t = Time is Seconds

(a) If the current applied was represented by i = Im Sin wt (a sinusoid), derive the expression for v_L :

(b) What would be the phase angle θ between the applied current and the voltage across the inductor?

Solution:

(a) $v = L \frac{di}{dt} = L \frac{d(Im \text{ Sin } wt)}{dt}$

$\frac{d}{dt} (Im \text{ Sin } wt) = wIm \text{ Cos } wt$

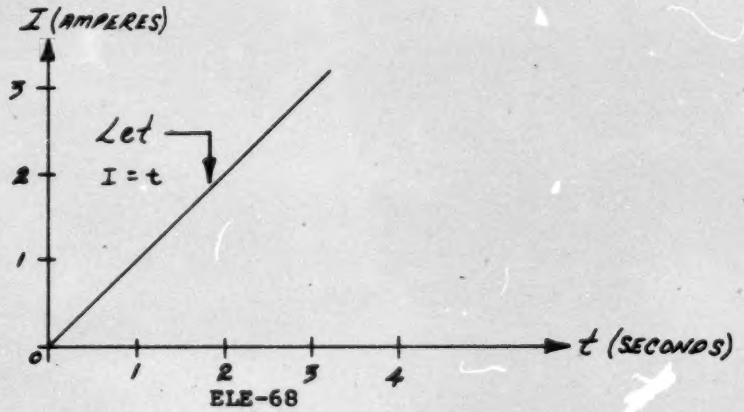
$v = wLIm \text{ Cos } wt$

(b) Since the current sinusoid is a sine function, and the voltage drop across the inductor is a cosine function,

$\theta = 90^\circ$ (voltage leading current)

B. Problems without Solutions

2. The instantaneous voltage on a capacitor is given by $Vc = \frac{1}{C}\int_0^t i\,dt$ where C = capacity in farads; i = instantaneous current in amperes and t = time in seconds. If a linearly increasing current of i = t amperes were applied to a capacitor of capacity C farads (as illustrated), derive an expression for Vc as a function of time.



Complete Solutions to B Problems

2. $$V_c = \frac{1}{C}\int_0^t 1\,dt = \frac{1}{C}\int_0^t (t)\,dt = \frac{1}{2C}\left[t^2\right]_0^t = \frac{1}{2C}t^2$$